B.Math. Hons. IInd year
Backpaper examination
Second semester 2017
Topology
Instructor : B.Sury
Answer any FIVE questions.

Q 1.
(i) Let $p : X \to Y$ be a map of topological spaces. Define what is meant by p being a quotient.
(ii) If $p : X \to Y$ is a quotient map and A is a closed subspace of X, then prove that $p : A \to p(A)$ is a quotient map.
(iii) Give an example to show that the product of two quotient maps need not be a quotient map.

Q 2.
(i) Prove that a product of finitely many connected spaces is connected.
(ii) Consider the product $\mathbf{R}^{\omega}$ of countably many copies of $\mathbf{R}$, under the box topology. Show that this is not connected;
Hint: Show that the subset U of bounded sequences and the subset V of unbounded sequences are both open in box topology.

Q 3. Prove that a compact subspace of a Hausdorff space is closed.

Q 4. Suppose X is a topological space with a countable open basis. Prove that every open covering of X has a countable subcollection which covers X. Further, deduce that X is separable.

Q 5. Recall that the K-topology is defined on $\mathbf{R}$ is defined as one whose basis consists of all $(a, b) \setminus K$ where $K = \{1/n : n > 0\}$ and (a, b) runs over open intervals. Prove that $\mathbf{R}$ with the K-topology is a Hausdorff space that is not regular.

Q 6. Let $f, g : X \to Y$ be continuous functions where Y is Hausdorff. Show that

$$\{x \in X : f(x) = g(x)\}$$

is closed in X.

Q 7. Let X be any path-connected space. Consider the cone of X defined as

$$CX = X \times [0,1]/\sim$$

where $(x,1) \sim (y,1)$ and $(x,0) \sim (y,0)$ for all $x, y \in X$. Prove that CX is contractible.